Redundancy and Ductility for Bridge Design

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My article in the Summer 2020 issue of *ASPIRE*® provided an introductory discussion on structural, load path, and internal redundancies. This article is the first of a series of articles that will continue to expand that discussion of redundancy. The series is intended to provide a context for discussions of different types of redundancies for concrete bridges that can be used when adopting new materials and technologies, as the art of concrete bridge design continues to advance. At the conclusion of the series, the resilience and robustness of concrete bridges will hopefully be clear to all of us. These attributes of concrete bridges, alongside their cost-effectiveness, architectural appeal, reduced maintenance costs, and durability, are the reasons bridge engineers and owners around the world have chosen, and continue to choose, concrete bridges as the preferred bridge solution. As we begin this series, it should also be noted that the topics covered in this article and the upcoming articles are not new; they are based on concepts that are well known by engineers designing concrete bridges.

Preamble

To facilitate this discussion, let us start with an example of a concrete beam that will introduce the key terminology used in this article and its relationship to the notation and concepts used in the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications.*[1] **Figure 1** shows a typical structural test conducted on a reinforced or prestressed (pretensioned or post-tensioned) concrete beam. To simplify our discussion, let us focus on a case where structural performance is governed by flexure. For such a test specimen, we can consider its flexural capacity from several perspectives. When using the AASHTO LRFD specifications to calculate the nominal flexural resistance M_n of such a beam, we simplify our calculations and typically make conservative assumptions by, for example, using a rectangular stress block (that is, the Whitney stress block) to represent the distribution of compressive stresses on the compression side, and by assuming a bilinear response for the reinforcement by ignoring strain hardening. For this example, we assume that the design compressive strength of concrete is 4 ksi, the beam is reinforced with Grade 60 deformed reinforcing bars, and the strain hardening behavior of the flexural reinforcement, which may be significant, is ignored. The experimental capacity M_{exp} of the member is determined based on the actual strength of concrete, which in this example is 4.5 ksi, somewhat higher than the specified strength; the actual yield strength of Grade 60 reinforcing bars, which in this case is 67 ksi and also somewhat higher than their nominal strength; and the strain hardening behavior of the flexural reinforcement is considered, with f_{ult} = 85 ksi. Therefore, the actual or experimental capacity M_{exp} is significantly higher than M_n. This "overstrength" although present in many, if not all, bridge applications—is a margin that is typically ignored in our design calculations. In other words, our designs are based on what we specify, rather than what is likely to occur. The nominal flexural resistance of the beam M_n is then multiplied by a resistance factor ϕ that is less than or equal to 1.0 to obtain the factored flexural resistance

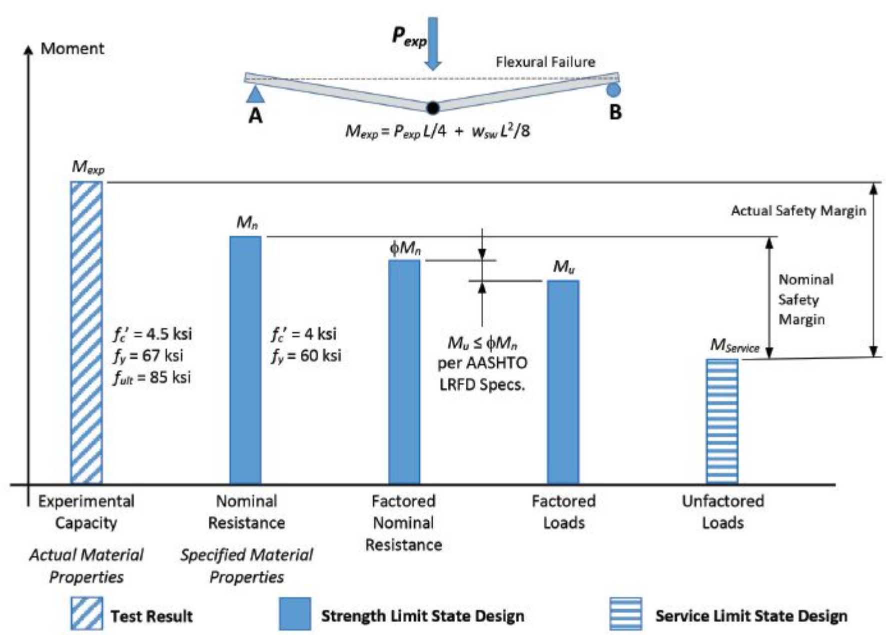


Figure 1. Design and experimental moments and associated safety margins for a reinforced or prestressed concrete beam. Note: f'_c = compressive strength of concrete; f_y = yield strength of flexural reinforcement; f_{ult} = ultimate strength of flexural reinforcement; L = span length; M_{exp} = experimental capacity; M_n = nominal resistance; $M_{Service}$ = moment due to unfactored (service) loads; P_{exp} = experimental load; w_{sw} = beam self weight; ϕ = resistance factor; ϕM_n = factored flexural resistance. Figure: Dr. Oguzhan Bayrak.

$M_r = \phi M_n$. In other words, the nominal resistance is reduced by the resistance factor to account for variability of material properties, structural dimensions, and other uncertainties. Also, the AASHTO LRFD specifications make explicit accommodations for brittle and ductile failure modes by assigning an appropriate ϕ factor for flexural resistance (see the discussion related to variable resistance factors later in this article). As explained in the following section, material properties and the resistance factor ϕ are not the only sources of safety margin in our designs.

Vehicular Live Load

For bridge design, engineers use the AASHTO LRFD specifications' HL-93 vehicular live load, which consists of the design truck or design tandem and the design lane load. This vehicular live load is a notional load and does not represent a specific type of vehicle (see the LRFD articles in the Summer 2009 and Fall 2009 issues of *ASPIRE* for a more complete discussion of the development and significance of the HL-93 vehicular live load). As described in AASHTO LRFD Article C3.6.1.2.1, the load was "developed as a notional representation of shear and moment produced by a group of vehicles routinely permitted on highways of various states under 'grandfather' exclusions to weight laws."

As traffic patterns change, the vehicular live load model continues to be monitored and evaluated using weigh-in-motion measurements taken over past decades.

Safety Margin

While the safety margin for bridges is not explicitly addressed in the AASHTO LRFD specifications, the concept is incorporated by requiring the moment M_u from factored load effects to be less than the factored resistance ϕM_n, as stated in a general form in AASHTO LRFD Article 1.3.2.1. A margin of safety is achieved in the context of load and resistance factor design methodology by calibrating the load and resistance factors for the strength-limit state using a statistical approach to ensure a "probability of exceedance of 2/10,000 during the 75 year design life of the bridge" (AASHTO LRFD Article C1.3.2.1), which corresponds to a target reliability index of $\beta = 3.5$ (see the LRFD article in the Winter 2007 issue of *ASPIRE* for more information on the calibration approach). This concept will be discussed further in a future article in this series.

Design for the strength-limit state, according to AASHTO LRFD Article 1.3.2.4, is intended "to ensure that strength and stability, both local and global, are provided to resist the specified statistically significant load combinations that a bridge is expected to experience in its design life." Using the terminology of the AASHTO LRFD specifications for flexural design, the demand M_u from the factored load effects must not exceed the factored resistance ϕM_n. Load factors for the strength-limit state are greater than 1.0, unless the use of a smaller load factor results in a more severe effect for a particular load combination. When designing the typical reinforced or prestressed concrete beam of Fig. 1 for the Strength I limit state load combination, we would use a dead load factor of 1.25 and a live load factor of 1.75. To simplify the discussion for this beam, other types of loads and the multiple presence of live loads will not be considered. As illustrated in Fig. 1, by requiring ϕM_n to be greater than or equal to M_u, a nominal safety margin is provided. This safety margin is not explicitly discussed in the AASHTO LRFD specifications, nor is it "AASHTO-sanctioned" terminology. Furthermore, this safety margin is not constant because it depends on the dead load (DL) to live load (LL) ratio (DL/LL) and other factors.

Design for the service-limit state, according to AASHTO LRFD Article 1.3.2.2, is intended to provide "restrictions on stress, deformation, and crack width under regular service conditions." The service-limit state represents the day-to-day operation of the bridge, usually with the maximum expected vehicular load as discussed earlier, so service-limit-state design provisions address serviceability criteria. Load factors for service-limit states are 1.0 (AASHTO LRFD Article 1.3.2.1), except in special situations, like the Service III limit state for prestressed concrete design. Initially, service-limit-state provisions were experience based and were not calibrated as had been done for the strength-limit states. Also, the consequence of exceeding a service-limit-state requirement is damage with a potential for decreased performance or service life rather than excessive yielding and cracking, as is generally the case when strength-limit-state requirements are exceeded.

The concept of a safety margin for the service-limit state is difficult to develop, but it is instructive to compare service loads to the nominal resistance or experimental capacity as shown in Fig. 1 to obtain a functional or operational feel for the safety margin for an element. For the concrete beam tested in flexure, as shown in Fig. 1, the actual safety margin between the service load $M_{Service}$ and the experimental capacity M_{exp} is typically greater than the nominal safety margin between the service load and the nominal capacity M_n. While this discussion has been focused on flexural behavior, analogous discussions can be formulated for shear and other behavioral modes.

Structural Redundancy

With this background, we can now move to the discussion of structural redundancy. To simplify our discussion, let us focus on actual behavior and nominal capacities. Certainly, as previously discussed, factored loads and resistances in compliance with the AASHTO LRFD specifications, which set the minimum standards employed in bridge design, will provide sufficient safety margins. More sophisticated analyses resulting in better estimates of strength are permitted but are not mandated. However, using such methods will typically reduce the safety margin, with due credit given to sophisticated analyses.

Structural redundancy is directly related to structural indeterminacy. A simply supported beam will form a collapse mechanism after the formation of one plastic hinge (**Fig. 2a**), whereas a continuous beam will require the formation of two plastic hinges (**Fig. 2b**) before a collapse mechanism can form. This means that when the yielding of flexural reinforcement occurs at midspan, the load effects can be redistributed within an indeterminate structure from the span with the initial hinge to the

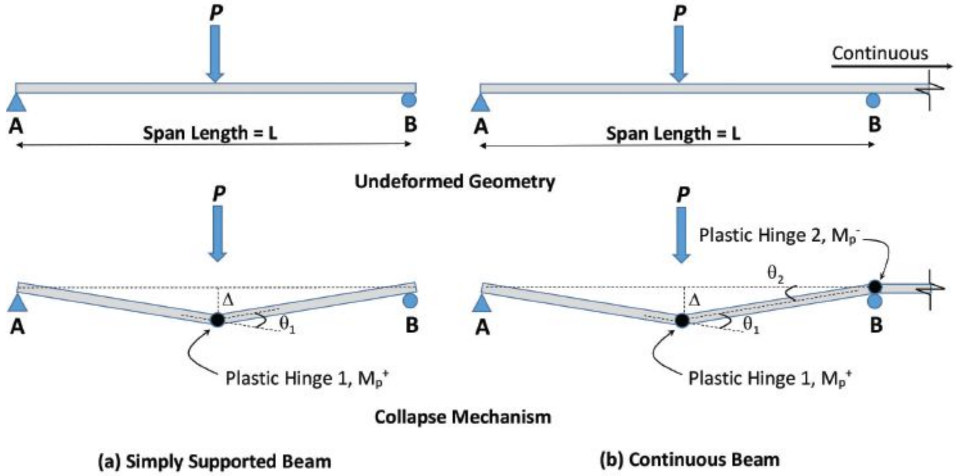


Figure 2. Collapse mechanisms for (a) simply supported beam and (b) continuous beam. Note: L = span length; M_p^+ = positive plastic moment; M_p^- = negative plastic moment; P = applied load; Δ = deflection at plastic hinge 1; Δ_1 = rotation at plastic hinge 1; θ_2 = rotation at plastic hinge 2. Figure: Dr. Oguzhan Bayrak.

remaining portion of the member. While not typically used in designs, the collapse load can be calculated using energy equilibrium equations. For the example, in Fig. 2, external work is equal to the product of the applied load and the deflection, that is $P\Delta$. Internal work is equal to the product of the plastic moment and the rotation, which is $M_p^+ \theta_1$ for the simply supported beam, and $M_p^+ \theta_1 + M_p^- \theta_2$ for the continuous beam. Because the load is acting at midspan, $\theta_2 = 0.5\theta_1$, and further, $\theta_2 = 2\Delta/L$, where L is the span length. Combining these expressions by equating the internal work to external work, we can calculate the collapse load for each case. That is, for a given geometry we can relate the collapse load P to M_p^+ for the simply supported beam, and to M_p^+ and M_p^- for the continuous beam.

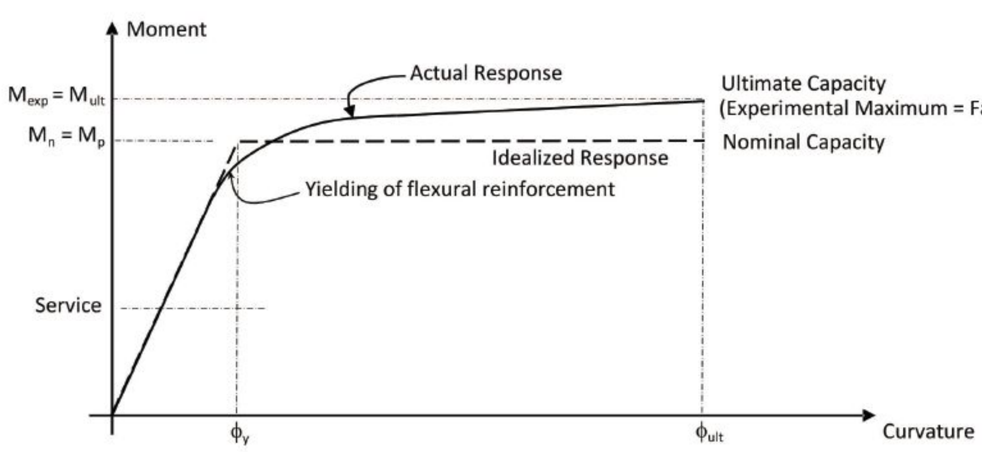
The concept of structural redundancy also highlights an important aspect of structural behavior that was not addressed in the calibration of the AASHTO LRFD specifications: the behavior of a structural system rather than a structural component. The calibration at the strength-limit state for the AASHTO LRFD specifications was based entirely on the performance of structural elements. However, if the interaction of elements in a structure is considered, there is often additional capacity that can be achieved as loads can be redistributed to adjoining elements when one element has reached its capacity. However, these benefits are not easily quantified, and much work remains before structural system behavior can be considered in the design specifications.

Ductility of Cross Section

To realize the benefits of structural redundancy, the member must have adequate ductility at the hinge locations. Evaluating the member ductility at the plastic hinge locations needs to be performed at the sectional level. For example, let us assume the beams shown in Fig. 2 are reinforced concrete beams detailed such that they possess ample shear strength and their response will be governed by flexural yielding. In this case, we can use sectional analysis software programs that perform layered-section analyses and determine the response of the beam section. **Figure 3** shows a typical moment-curvature response and its idealized form. While this figure represents positive bending response, such as at hinge 1 in Fig. 2, a similar response can be evaluated for negative bending, such as at hinge 2 in the continuous beam in Fig. 2.

In Fig. 3, the idealized form of the moment-curvature response has slightly lower moment capacity $M_n = M_p$ than the actual moment capacity M_{ult}. On the deformation side (the horizontal axis), the idealized yield curvature ϕ_y and ultimate curvature ϕ_{ult} are identified for the calculation of curvature ductility. The ratio of ϕ_{ult} to ϕ_y is known as the curvature ductility factor. This factor denotes the ability of the section to undergo plastic deformation in a stable manner, that is, without losing capacity. The moment-curvature relationship shown in Fig. 3 can be used to obtain the moment-rotation relationship, which can also be used as a measure

Figure 3. Actual and idealized moment-curvature responses for a reinforced concrete beam. Note: M_{exp} = experimental capacity; M_n = nominal resistance; M_p = plastic moment; M_{ult} = actual ultimate capacity; ϕ_{ult} = curvature at ultimate capacity (failure); ϕ_y = idealized yield curvature. Figure: Dr. Oguzhan Bayrak.



of overall ductility of the structure. To use this approach, the length of the plastic hinge must be known, or conservatively assumed. Mattock made a series of recommendations in this regard.[2] More recently, Bae and Bayrak analyzed a series of different conditions for columns and provided an expanded set of guidelines that reflect lessons learned since the 1960s.[3] When applying this concept to girders, it is important to appreciate that a traditional plastic hinge that develops in a well-confined region of a reinforced concrete column and one that develops in a beam (that is, one or a few wide cracks leading to yielding of reinforcement that crosses those cracks) are quite different from one another. The moment-rotation relationship for the member whose moment-curvature is shown in Fig. 3 would have a very similar shape to the moment-curvature behavior depicted in Fig. 3, but with the horizontal axis representing rotation. With this information, rotational ductility of the plastic hinge regions can now be explored. To take advantage of structural redundancy, a member should be detailed to permit the formation of a sufficient number of hinges and allow significant deflection that will signal impending failure. For details typically seen in our concrete bridges, it is expected that redundancy due to continuity provided at support B as in Fig. 2b will increase the collapse load and result in additional warning of impending failure.

Deformation Capacity

Detailing of the beam—material properties for the concrete, longitudinal reinforcement, and presence of confinement reinforcement (transverse reinforcement)—will determine the shape of the moment-curvature or moment-deflection curve, and therefore the ductility of the response. For reinforced concrete, plastic deformations can largely be attributed to the yielding of flexural tension reinforcement. The deformation capacity of the flexural tension reinforcement has a significant effect on the curvature ductility and therefore the rotational capacity of the section. For example, ASTM A615 Grade 60 reinforcing bars rupture at a minimum strain of 7% to 9%. ASTM A706 Grade 60 reinforcing bars are more deformable and rupture at a strain level of 10% to 14%. These high-rupture strains indicate that members reinforced with these materials can experience large rotations after cracking, thereby providing ample warning prior to failure as long as a compression failure is avoided.

Now we will consider prestressing strands and their effect on the behavior of prestressed concrete girders. Typical volume changes that prestressed concrete members experience due to creep and shrinkage are large enough to render the use of conventional reinforcement impractical in prestressed concrete applications. It is for this reason that the invention of high-strength steel wires made prestressed concrete viable. Since the first prestressed concrete girder bridge in the United States (Walnut Lane Memorial Bridge in Philadelphia, Pa.) was constructed in 1950, the strength of prestressing reinforcement (wires, post-tensioning bars, and strands) has gradually increased over the years. Today, the most common grade for prestressing strands is Grade 270. While this product was first produced using stress-relieving techniques, a strain-tempering process was subsequently introduced, and today low-relaxation strands are the industry standard. The minimum elongation capacity of Grade 270 seven-wire strands is 3.5% according to ASTM A416. This value is substantially smaller than the deformation capacity of Grade 60 deformed reinforcing bars, as previously discussed. Even with the reduced material deformation capacities of prestressing strand, prestressed concrete beams have been routinely used in our bridges, it has been demonstrated repeatedly that prestressed concrete girders have sufficient ductility to form wide cracks and significant deflections that signal impending failure.

This leads to a discussion of how much deformation capacity is needed in our bridges. While the answer to this question is somewhat subjective, it is commonly accepted that we would like to see sufficient levels of inelastic deformation (deflection) and associated cracking in our bridges to provide warning as they are loaded to failure. Based on decades of structural tests, the deformation capacity displayed by typical prestressed concrete bridges has been shown to be more than adequate.

As discussed earlier, the AASHTO LRFD specifications are focused on providing adequate strength but aspects of deformation capacity and ductility are included in the design provisions, which are intended to ensure yielding and cracking. However, the specificactions permit members with limited ductility based on the use of a variable resistance factor as shown in **Fig. 4**. Ductile girders are assumed to be governed by

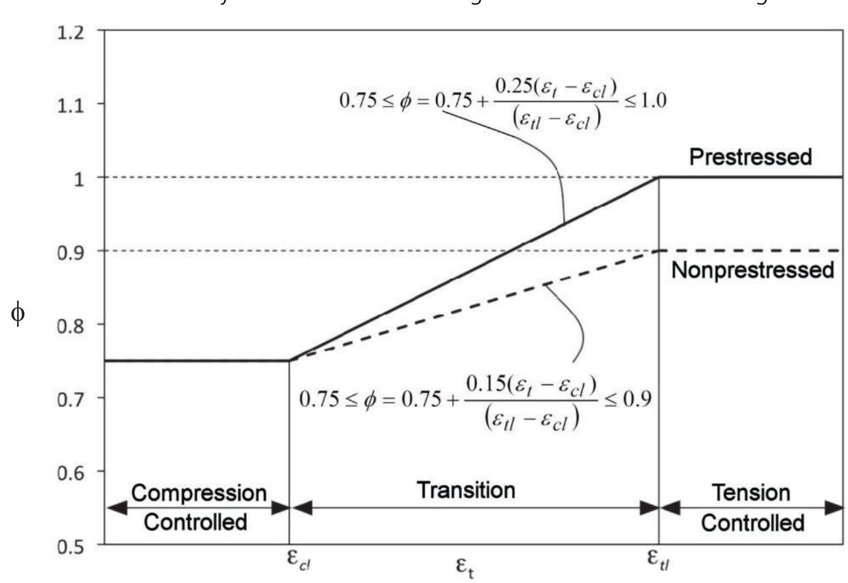


Figure 4. Variation of resistance factor ϕ with net tensile strain ε_t for prestressed and nonprestressed elements as behavior transitions from a tension-controlled section to a compression-controlled section. Figure: *AASHTO LRFD Bridge Design Specifications*, 9th ed., Figure C5.5.4.2-1.

tension-controlled behavior in which the tension reinforcement yields. However, the current design provisions also allow compression-controlled designs through the use of a lower resistance factor, as shown in the figure. While compression-controlled designs are very rare in flexural members, the specifications do allow them, albeit with lower resistance factors. With the introduction of some new materials as flexural reinforcement, it may become more common to see designs where the member capacity is compression controlled. The appropriate implementation of this concept will be discussed in future articles in this series.

The AASHTO LRFD specifications also require that members have a minimum quantity of reinforcement to avoid failure as soon as a member develops a flexural crack. These provisions, which are found in AASHTO LRFD Article 5.6.3.3, require that the minimum amount of prestressed and nonprestressed tensile reinforcement be adequate to develop factored flexural resistance ($M_r = \phi M_n$) greater than or equal to the lesser of 1.33 times the factored moment M_u and the cracking moment M_{cr}. The calculation of M_{cr} using AASHTO LRFD Eq. 5.6.3.3-1 includes factors that take into account the variability in the flexural cracking strength of concrete, the variability of prestress, and the ratio of nominal yield stress to ultimate stress for the flexural reinforcement. So in this situation, members must be designed to provide a significantly increased nominal capacity so the probability of a nonductile (brittle) failure is very low.

Conclusion

This article is the first of a series of articles on redundancy and ductility. It has provided background on the basic design approach in the AASHTO LRFD specifications and a further discussion of the concept of structural redundancy. Subsequent articles will continue discussions of different types of redundancies for concrete bridges to provide background information that we can use in adopting new materials and technologies as bridge design continues to evolve.

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