Design for Torsional Effects

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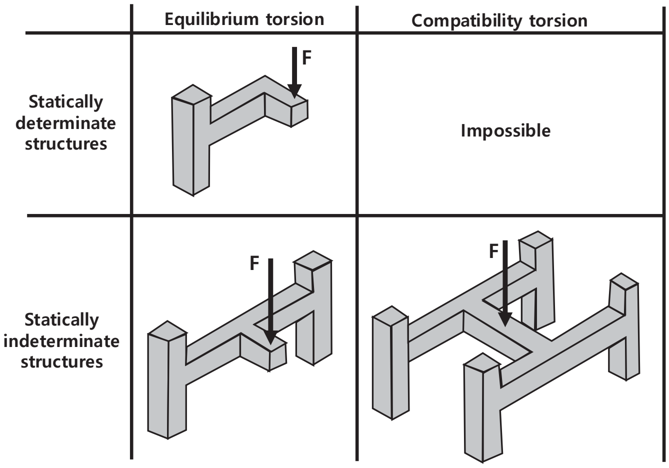
Complex geometry, as well as complex loading conditions that we see in our bread-and-butter bridges, can result in torsional moments (or torques) being applied to concrete components. One of the topics that generates questions from our bridge design community involves requirements in the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*[1] that apply to torsional designs. This article is devoted to providing a succinct discussion of this topic.

Let us begin our discussion by considering under what circumstances the AASHTO LRFD specifications deems torsional moments to be negligibly small. According to Eq. (5.7.2.1-3) in those specifications, if the factored torsional moment T_u is less than 25% of the factored torsional cracking moment $0.25\phi T_{cr}$, torsional effects do not need to be investigated. In such circumstances, torsional effects cause only a very small reduction in shear capacity or flexural capacity. More specifically, the following requirements of the AASHTO LRFD specifications can be used as a starting point to assess whether we need to consider torsion in structural designs:

> *Torsional effects shall be investigated where:*
>
> $$T_u > 0.25\phi T_{cr} \qquad (5.7.2.1\text{-}3)$$
>
> *For solid shapes:*
>
> $$T_{cr} = 0.126K\lambda\sqrt{f'_c}\,\frac{A_{cp}^2}{p_c} \qquad (5.7.2.1\text{-}4)$$
>
> *For hollow shapes:*
>
> $$T_{cr} = 0.126K\lambda\sqrt{f'_c}\,2A_o b_e \qquad (5.7.2.1\text{-}5)$$
>
> *in which:*
>
> $$K = \sqrt{1 + \frac{f_{pc}}{0.126\lambda\sqrt{f'_c}}} \le 2.0 \qquad (5.7.2.1\text{-}6)$$
>
> *where:*
>
> T_u = *applied factored torsional moment (kip-in.)*
> T_{cr} = *torsional cracking moment (kip-in.)*
> A_o = *area enclosed by the shear flow path, including any area of holes therein (in.^{2})*
> A_{cp} = *area enclosed by outside perimeter of concrete cross-section (in.^{2})*
> p_c = *length of outside perimeter of the concrete section (in.)*
> f'_c = *compressive strength of concrete for use in design (ksi)*
> f_{pc} = *unfactored compressive stress in concrete after prestress losses have occurred either at the centroid of the cross-section resisting transient loads or at the junction of the web and flange where the centroid lies in the flange (ksi)*
> b_e = *effective width of the shear flow path taken as the minimum thickness of the exterior webs or flanges comprising the closed box section (in.). b_e shall be adjusted to account for the presence of ducts.*
> ϕ = *resistance factor specified in Article 5.5.4.2*
> λ = *concrete density modification factor as specified in Article 5.4.2.8*
>
> *b_e defined above shall not exceed A_{cp}/p_c, unless a more refined analysis is utilized to determine a larger value.*
>
> *The effects of any openings or ducts in members shall be considered. K shall not be taken greater than 1.0 for any section where the stress in the extreme tension fiber, calculated on the basis of gross section properties, due to factored load and effective prestress force after losses exceed $0.19\lambda\sqrt{f'_c}$ in tension.*
>
> *When calculating K for a section subject to factored axial force, N_u, f_{pc} shall be replaced with $f_{pc} - N_u/A_g$. N_u shall be taken as a positive value when the axial force is tensile and as a negative value when it is compressive.*

For cases in which torsional effects are significant, the next step in design involves a thorough understanding of equilibrium and compatibility torsion. This understanding has design implications, as will be discussed in the later sections of this article. In statically determinate structures, consideration of torsional effects becomes particularly significant. To study equilibrium torsion, let us consider, for example, a hammerhead pier supporting spans of different lengths up station and down station. The beam reactions applied on the hammerhead pier cap could differ significantly and subject the cap to torsional effects, such as shown in the upper left corner of **Fig. 1**.

Figure 1. Illustrations of equilibrium and compatibility torsions. Source: *AASHTO LRFD Bridge Design Specifications* Fig. C5.7.2.1-1.[1]



If the applied torque exceeds the threshold torque 0.25ϕT_{cr}, designing the cap for torsional effects would be mandatory. Importantly, the factored torsional resistance shall be greater than or equal to the factored torsional moment acting on the member (that is, $\phi T_n \geq T_u$). At the strength limit state, failing to provide sufficient torsional strength would result in torsional cracking, and the torsional cracks would establish a torsional hinge and allow the cantilever portion of the cap to rotate freely. The plastic mechanism that forms in this example would lead to structural collapse.

In a statically indeterminate case, we may or may not be dealing with equilibrium torsion. For the case shown at the bottom left corner of Fig. 1, formation of torsional hinges at the faces of the column would lead to a plastic mechanism and total loss of equilibrium (that is, collapse). In this case, factored torsional resistance must be greater than or equal to the factored torque, like the previous case we discussed. The details of those calculations are discussed in upcoming sections of this article.

Let us next examine the compatibility torsion case that is shown as the bottom right corner of Fig. 1. Compatibility torsion is the type of torsion that is rooted in keeping the deformations of adjacent elements compatible in their deformed state. Let us envision a case in which torsional hinges form at the faces of the supporting columns. In this scenario, the twist experienced in the ends of the beams will relieve the applied torque. Therefore, rather than resisting the full factored torque applied at the column faces, "twisting" of the beam ends can be permitted, and the beams can be relieved from the applied torsional moments. To this end, commentary for Article 5.7.2.1 of the AASHTO LRFD specifications states:

> *It is not necessary to design for compatibility torsion as long as the other load paths are properly designed for the redistributed forces. Consideration should be given to the aesthetic issues that may be caused by cracking that could be associated with not designing for compatibility torsion.*

Furthermore, the code portion of Article 5.7.2.1 states:

> *In a statically indeterminate structure where significant reduction of torsional moment in a member can occur due to redistribution of internal forces upon cracking, the applied factored torsional moment at a section,* T_u*, may be reduced to* ϕT_{cr}*, provided that moments and forces in the member and in adjoining members are adjusted to account for the redistribution.*

In the aforementioned case, we must expect to see torsional cracks to allow for redistribution of the forces and create an opportunity for the loads to look for alternative load paths. This redistribution of forces is coupled with the need to design the section for ϕT_{cr}.

To complete this discussion, let us take a look at provisions in the AASHTO LRFD specifications for calculating torsional resistance, for cases in which we must perform a complete torsional design. Article 5.7.3.6.2 indicates:

> *The nominal torsional resistance shall be taken as:*
>
> $$T_n = \frac{2A_o A_t f_y \cot\theta}{s}\lambda_{duct} \qquad (5.7.3.6.2\text{-}1)$$
>
> *where:*
>
> A_o = *area enclosed by the shear flow path, including any area of holes therein (in.*^{2}*)*
>
> A_t = *area of one leg of closed transverse torsion reinforcement in solid members, or total area of transverse torsion reinforcement in the exterior web and flange of hollow members (in.*^{2}*)*
>
> θ = *angle of inclination of diagonal compressive stresses as determined in accordance with the provisions of Article 5.7.3.4 with the modifications to the expressions for* v *and* V_u *herein (degrees)*
>
> s = *spacing of transverse reinforcement measured in a direction parallel to the longitudinal reinforcement (in.)*
>
> λ_{duct} = *shear strength reduction factor as defined in Eq. 5.7.3.3-5*

Note that the transverse steel area for torsion A_t is defined differently from the transverse steel area for flexural shear A_v. A common error is to confuse the two definitions.

Torsion increases the demand on shear reinforcement. In addition, torsion increases the demand on longitudinal reinforcement. To that end, Article 5.7.3.6.3 in the AASHTO LRFD specifications states:

> *The provisions of Article 5.7.3.5 shall apply as amended, herein, to include torsion. At least one bar or tendon shall be placed in the corners of the stirrups.*
>
> *The longitudinal reinforcement in solid sections shall be proportioned to satisfy Eq. 5.7.3.6.31:*
>
> $$A_{ps}f_{ps} + A_s f_y \geq \frac{|M_u|}{\phi d_v} + \frac{0.5N_u}{\phi} + \cot\theta\sqrt{\left(\left|\frac{V_u}{\phi} - V_p\right| - 0.5V_s\right)^2 + \left(\frac{0.45p_h T_u}{2A_o\phi}\right)^2} \qquad (5.7.3.6.3\text{-}1)$$
>
> *In box sections, longitudinal reinforcement for torsion, in addition to that required for flexure, shall not be less than:*
>
> $$A_\ell = \frac{T_n p_h}{2A_o f_y} \qquad (5.7.3.6.3\text{-}2)$$
>
> *where:*
>
> p_h = *perimeter of the centerline of the closed transverse torsion reinforcement for solid members, or the perimeter through the centroids of the transverse torsion reinforcement in the exterior webs and flanges for hollow members (in.)*
>
> A_ℓ *shall be distributed around the outermost webs and top and bottom slabs of the box girder.*

In summary, torsional effects impose additional demands, and therefore require additional reinforcement, both in longitudinal and transverse directions. This conclusion can be best visualized in the space truss model (also known as the tubular truss model) shown in **Fig. 2**. A given torsional moment results in an increased demand on one face of a typical structural component, thereby increasing the demand on the torsional reinforcement on that face. However, typical structural components are reinforced symmetrically in view of the force

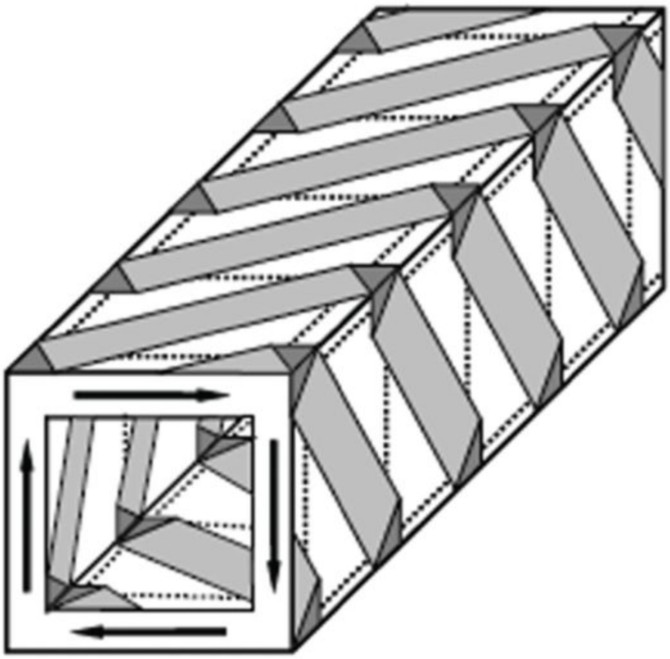


Figure 2. Strut-and-tie model of a space truss subjected to a torsional moment. Source: *Anchorage of Headed Reinforcement in CCT Nodes*.[2]

flow that is discussed in Fig. 2. Notably, in the terminology of the AASHTO LRFD specifications, all the nodes shown in this strut-and-tie model (STM) are smeared nodes. What is very clear is that the formation of a helical compression field in three-dimensional space requires ties in the longitudinal and transverse directions. Notably, the previously discussed torsional design requirements from the AASHTO LRFD specifications are based on the model in Fig. 2. In other words, should we choose to design for torsional effects using an STM, we would reach a design equivalent to what is explicitly required by the previously discussed provisions.

Understanding the implications of compatibility and equilibrium torsions is a key starting point for torsional design. Comparing the applied factored torsional moments to a member to the threshold torque is a necessary second step. Following these initial steps, additional decisions can be made to complete the structural design for torsional moments.

References

1. American Association for State Highway and Transportation Officials (AASHTO). 2020. *AASHTO LRFD Bridge Design Specifications*. 9th ed. Washington, DC: AASHTO.
2. Thompson, M.K., M.J. Young, J.O. Jirsa, J.E. Breen, and R.E. Klingner. 2003. *Anchorage of Headed Reinforcement in CCT Nodes*. Center for Transportation Research Report 1855-2. Austin, TX: University of Texas at Austin.

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