Concrete Segmental Bridges—

Preliminary Design Approximations for Creep Redistribution, Post-Tensioning Secondary Moments, and Thermal-Gradient Stresses

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This article, which is the third in a series discussing preliminary design approximations for concrete segmental bridges, covers methods to estimate creep redistribution, post-tensioning secondary moments, and thermal-gradient stresses. For most loads, such as superimposed dead loads and live loads, moment demand can be determined through the use of a simple finite element model with a limited number of nodes and elements, and no consideration of time-dependent effects. Before the introduction of desktop computers, beam charts were used for this purpose. However, determining the moments and stresses covered in this article is a more time-consuming process that requires more-precise modeling; therefore, simplified methods and approximations are helpful for preliminary designs.

For instance, the determination of the moment demands from creep and shrinkage requires a complex time-dependent—and time-consuming—model, which is typically developed in final design. Therefore, a simplified method for determining redistribution moments is desirable for preliminary design. Post-tensioning secondary moments are typically treated as a demand and are dependent on the number, size, and profile of the selected tendons in the bridge. Secondary moments can be calculated with a finite element model that includes the modeling of post-tensioning tendons in final design. However, a simple and noniterative method for determining secondary moments is desirable for preliminary design. Similarly, the computation of stresses due to nonlinear thermal gradients is time-consuming, and a simple approximate calculation method is desirable for preliminary design.

Creep Redistribution

When a superstructure is erected in a static scheme different from the final static scheme of the bridge, the forces in the superstructure will tend to redistribute due to creep. To help understand the redistribution effect, consider the structure constructed using the balanced-cantilever method shown in **Fig. 1**. After the cantilevers are erected, there are large negative moments over the piers and no positive moments at midspan due to the self-weight of the cross section. As shown in Fig. 1, cantilever stresses have higher levels of compression in the bottom fiber than in the top fiber. Therefore, creep of the concrete will cause the cantilever to deflect downward over time. If the cantilever is free, there is a corresponding rotation at its tip. However, if continuity has been established between cantilevers, the rotation of the ends of the cantilevers is restrained. As seen in Fig. 1, the moments that develop because of this restraint are positive. Therefore, for this balanced-cantilever example, positive moments develop at midspan and the negative moments over the piers are reduced. The net effect is a redistribution of the self-weight moments, with the moment diagram shifting downward (**Fig. 2**).

In general, for any structure, there is a self-weight moment diagram (and corresponding set of coincident forces for every degree of freedom) that occurs at the end of construction. Historically, this moment diagram has been termed the S_1 state. This S_1 moment diagram is dependent on the geometry and properties of the structure (span length, cross-sectional properties, self-weight, and so on), as well as the sequence and methods used to construct the structure.

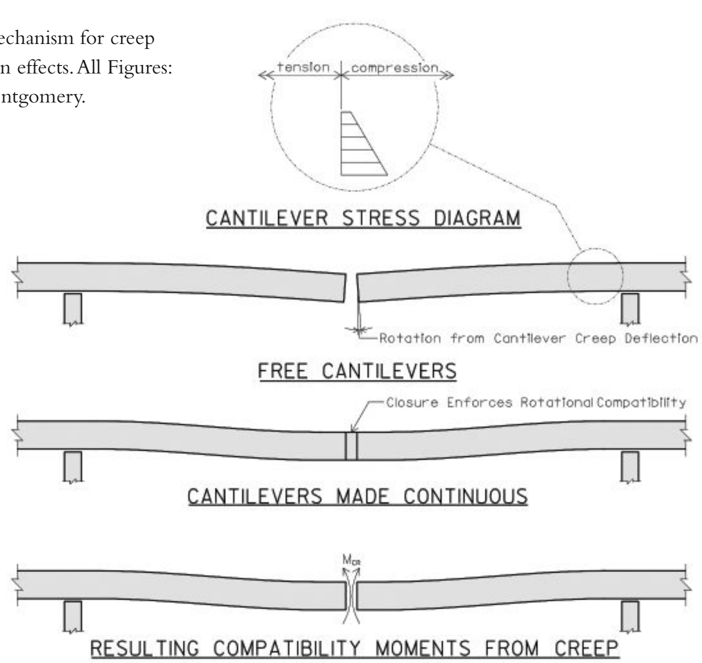

Figure 1. Mechanism for creep redistribution effects. All Figures: R. Kent Montgomery.

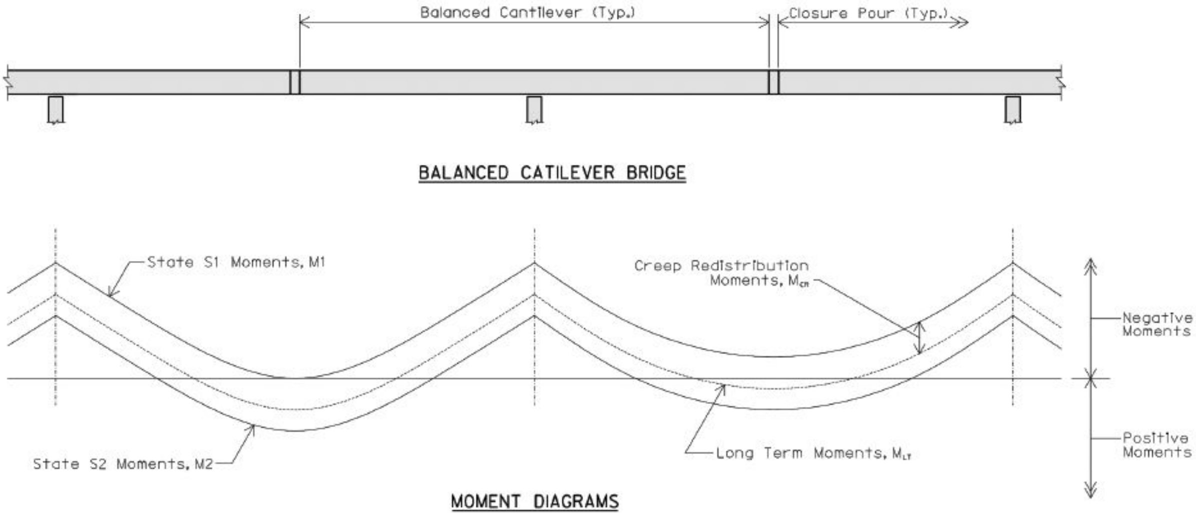

Figure 2. Creep redistribution in a concrete segmental bridge constructed using the balanced-cantilever method.

A different self-weight moment diagram would apply if the structure were analyzed as continuous for self-weight loading (as if the entire structure were built on falsework). This moment diagram is termed the continuous state or, historically, the S_2 state. As a general rule, concrete creep will cause the moment diagram to shift from the S_1 state toward the S_2 state as follows:

$$M_{LT} = M_1 + M_{CR}$$

where

M_{LT} = long-term moment
$M_{CR} = (1 - e^{-\psi})(M_2 - M_1)$ = creep redistribution moment
M_1 = S_1 state moment
M_2 = S_2 state moment
ψ = creep coefficient

Typically, the moment diagram never fully reaches the S_2 state but is instead somewhere between the S_1 and S_2 states. For preliminary design, it is reasonable to make the following assumptions:

- $(1 - e^{-\psi}) = 0.0$ for creep redistribution moments at the end of construction
- $(1 - e^{-\psi}) = 0.5$ to 0.7 for long-term creep redistribution moments

Figure 2 illustrates the use of this concept.

Note that although reference has been made to self-weight moments in the discussion about redistribution, the amount of post-tensioning and the point in time when a member is tensioned influence the amount of redistribution that occurs. For example, if additional cantilever post-tensioning is used, the compressive stress diagram in the cantilever becomes more uniform between the top and bottom fibers. Therefore, creep will produce more axial contraction but less downward deflection. As discussed earlier, it is the downward deflection that leads to the redistribution effect; therefore, adding cantilever post-tensioning reduces the amount of creep redistribution for the example shown in Fig. 2. However, the assumptions made earlier are usually adequate for preliminary design.

Other load effects, including barriers, wearing surface, live load, and temperature effects (uniform temperature and temperature gradient) are typically analyzed for the continuous structure and, as such, are not subject to the same time-dependent considerations.

Secondary Moments

Post-tensioning secondary moments arise in statically indeterminate structures due to restrained deformations from the post-tensioning. The secondary moments most relevant to this discussion are caused by restrained rotations. For example, a simple-span bridge is free to rotate at the ends of the span due to the primary post-tensioning forces. However, if the bridge is continuous, the ends of the span are not free to rotate unrestrainedly, and secondary post-tensioning moments develop. Note that primary post-tensioning forces are those applied by the prestressing without any restraints:

$$P_p = P_{pt}$$
$$M_p = P_{pt} \times e$$

where

P_p = primary post-tensioning axial force
P_{pt} = applied post-tensioning axial force
M_p = primary post-tensioning moment
e = tendon eccentricity

Forces developed due to restraint reactions are termed secondary forces. For calculation of stresses, the sum of these forces should be used. For the strength limit state, the secondary moments are treated as demands and the prestressing itself is used in capacity calculations.

The secondary moments for any tendon are dependent on the tendon length, profile, and position in the bridge. Therefore, analysis programs are the best option to exactly calculate secondary moments. However, the process of defining tendons in such a program to arrive at a post-tensioning layout would be iterative and time-consuming. The assumptions presented herein allow for simple, quick calculations with enough accuracy for preliminary design.

An important concept, without getting into detailed calculations, is that secondary moments are proportional to free-end rotations (rotations without restraints). Free-end rotations are then proportional to the area above or below the neutral axis for the primary moment diagram along the span (classic moment-area theorem). Experience has shown that some simple assumptions about the magnitude of secondary moments as a percentage of primary moments are sufficiently accurate for determining a preliminary post-tensioning layout to advance to final design.

For typical concrete segmental bridges constructed by the span-by-span method, it is reasonable to assume that the secondary moments in interior spans are 50% of the positive primary moment from all tendons. In other words, for each interior span, the secondary moment in the span is positive and equal to 50% of the maximum primary moment from all tendons. This assumption holds true when the span lengths in a unit are roughly equal (±20%) and the post-tensioning layout consists primarily of draped tendons anchoring in the pier or expansion diaphragms at the ends of each span and deviating in the span as shown in **Fig. 3**. The secondary moment diagram can be assumed to be constant across interior spans and decrease to zero at the free end of the unit (across the end spans). Therefore, the secondary moments at critical locations in an end span are less than the secondary moments in interior spans and can be determined by linear interpolation.

Note that the locations of the deviation diaphragms influence the magnitude of the secondary moments. Figure 3 shows that the closer the deviation diaphragms are to the pier diaphragms (that is, the further apart the deviation diaphragms are), the greater the area of the primary moment diagram below the neutral axis and, hence, the greater the secondary moments will be. However, the deviation diaphragms must be far enough apart such that the straight run of the tendons in the lowest position between deviation diaphragms captures the moment diagrams for load demands, including the effects of creep. (Note that creep redistribution is small for span-by-span bridges.) Spacing the deviation diaphragms approximately one-quarter of the span length apart is typically optimal (Fig. 3). Spacing them further apart increases the magnitude of the secondary moments and, therefore, decreases the efficiency of the tendons. Spacing them closer together does not capture the load-demand diagram as described previously. For interior spans, locating the deviation diaphragms so that they are centered in the span is usually optimal. For end spans, locating the deviation diaphragms so that they are centered on a location 40% of the span length from the end of the unit is usually optimal.

For constant-depth balanced-cantilever bridges, it is reasonable to assume that the secondary moments are 50% of the positive primary moment from all continuity tendons. (For each span, the secondary moment in the span is positive and 50% of the maximum primary moment from all continuity tendons.) This assumption applies to both bottom slab and draped span-by-span-style continuity tendons. Note that the bottom-slab tendons are not draped, and the primary moment diagram is a straight line at the maximum eccentricity. The area under the primary moment diagram is smaller for shorter bottom slab tendons, resulting in smaller secondary moments, and the area under the primary moment diagram is larger for longer bottom slab tendons, resulting in larger secondary moments (**Fig. 4**). The assumption that the secondary moments are 50% of the primary moments for bottom slab tendons is based on the average for all tendons. The secondary moment diagram can be assumed to be constant across interior spans and decrease to zero at the end of the unit across end spans. The same considerations for span-by-span bridges apply to locating the deviation diaphragms for draped tendons.

For variable-depth balanced-cantilever bridges, the location of the neutral axis is not a straight line across the span but instead follows a profile similar to the intrados profile (**Fig. 5**). The bottom slab tendon profile follows a profile just below the upper surface of the bottom slab, and it reasonable to assume that the secondary moment for these bottom slab tendons is positive and is 50% of the primary moment. Due to the profile of the neutral axis, the area under the neutral axis is smaller for draped tendons and is partially offset by the area above the neutral axis. Therefore, draped tendons

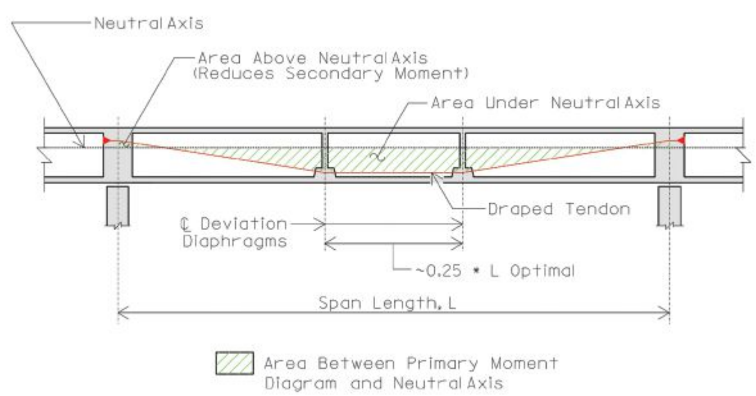

Figure 3. Draped post-tensioning tendon layout for a concrete segmental bridge constructed using the span-by-span method.

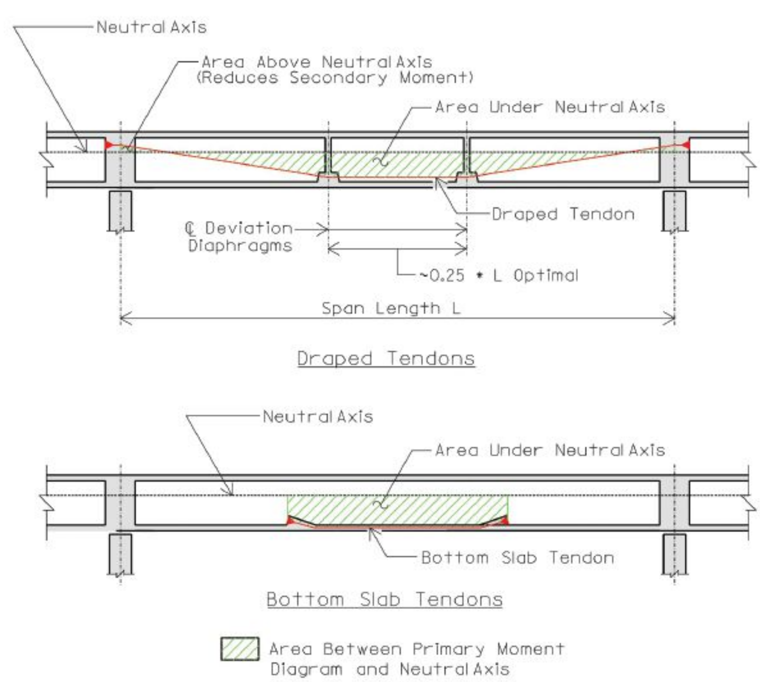

Figure 4. Continuity post-tensioning tendon layout for a constant-depth concrete segmental bridge constructed using the balanced-cantilever method.

are more efficient for variable-depth bridges than for constant-depth bridges, and it is reasonable to assume that the secondary moment for these draped tendons is positive and is 25% of the primary moment. The total secondary moment in the span is the sum of the secondary moments from the bottom slab and draped tendons. The secondary moment diagram can be assumed to be constant across all interior spans and to decrease to zero at the free end of the unit (across the end spans). The same considerations for span-by-span bridges apply to locating the deviation diaphragms for draped tendons.

For preliminary design, the number of required tendons can be estimated based on keeping stresses within the limiting stresses for the service limit state. Typically, end spans with external tendons represent the only situation in which the service limit state does not govern the amount of post-tensioning. A quick calculation of the moment capacity can determine whether the amount of post-tensioning in these spans needs to be increased.

For calculational purposes, the concept of tendon efficiency can be used to estimate the amount of post-tensioning. For example, for a tendon where the amount of secondary moment is 25% of the primary moment, the tendon is 75% efficient and the total stresses due to the post-tensioning can be calculated from the full axial force and 75% of the primary moment.

Temperature Gradient Stresses

Historically, a 10°C (18°F) positive linear temperature gradient was applied for design—with a positive gradient indicating that the top fiber is warmer than the bottom fiber. After 1989 and with the introduction of the American Association of State Highway and Transportation Officials' *Guide Specifications for Design and Construction of Segmental Concrete Bridges*,[1] nonlinear temperature gradients were specified, including a negative gradient. An accurate computation of stresses due to a nonlinear temperature gradient requires involved calculations; however, for preliminary design, simplifying assumptions can be made to ease the

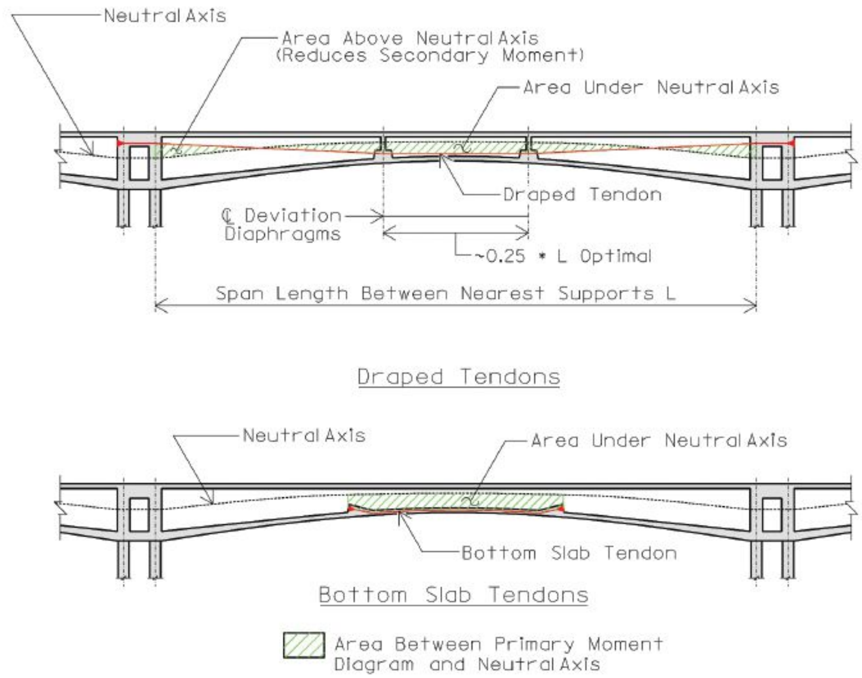

Figure 5. Continuity post-tensioning tendon layout for a variable-depth concrete segmental bridge constructed using the balanced-cantilever method.

computational burden. In concrete segmental bridges the stresses due to temperature gradients are much smaller than those from permanent loads and live loads; therefore, very simple assumptions yield adequate results.

First, the negative temperature gradient can be ignored for preliminary design. Negative temperature gradients typically only govern where the live-load stresses in the top slab are small (for example, near the free end of end spans), and refining the final design is relatively simple for these special regions.

For the positive temperature gradient, experience has shown that the following equivalent moment can be used for preliminary design:

$$M_{tg} = (\Delta T \times E \times I \times \alpha)/h$$

where

M_{tg} = equivalent moment for calculating stresses from a positive temperature gradient
ΔT = 15°F approximate temperature gradient for preliminary design
E = concrete modulus of elasticity
I = cross section moment of inertia
α = concrete coefficient of thermal expansion
h = overall depth of cross section

This thermal gradient moment is positive and roughly equivalent to the restraint moment that develops from a linear temperature gradient. The gradient of 15°F is a little less than the historically used 18°F linear gradient to account for the helpful bottom fiber compressive internal stresses from a positive nonlinear gradient. The full M_{tg} can be applied for interior spans, and the moment can be assumed to decrease to zero at the free end of the unit (across the end spans). For variable-depth spans, I and h can be taken at a section approximately 20% of the span length from the midspan.

Conclusion

The simplifications presented in this article can be used to calculate the amount of post-tensioning for concrete segmental bridges constructed by the span-by-span method and the amount of continuity post-tensioning for balanced-cantilever bridges. Methods to determine the amount of cantilever post-tensioning for balanced-cantilever bridges and the cross-sectional dimensions were discussed in previous articles in *ASPIRE*®. The remaining task for preliminary design is to lay out the individual post-tensioning tendons. This topic will be discussed in the next article in this series.

Reference

1. American Association of State Highway and Transportation Officials (AASHTO). 1989. *Guide Specifications for Design and Construction of Segmental Concrete Bridges*. Washington, DC: AASHTO.