Flexural Strength of Prestressed Concrete Bridge Beams with Composite Decks

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This article, the second of a two-part series on strain compatibility analysis, compares the methods for estimating the flexural strength of prestressed concrete bridge beams with composite decks specified in the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*.[1] Part 1 of the series, published in the Fall 2024 issue of *ASPIRE*®, introduced strain compatibility concepts and presented an example of strain compatibility analysis.

The AASHTO LRFD specifications provide two methods for estimating the flexural strength of composite prestressed concrete beams: a simplified approach based on the well-known rectangular stress distribution and the strain compatibility approach. The simplified approach gives reasonably accurate estimates for reinforced and prestressed concrete structural components, and when the concrete strength of all compression elements is similar for composite components. However, cast-in-place slabs on precast, prestressed concrete beam sections typically have different concrete strengths for the slab and beam. Using the simplified approach for these types of sections, the estimated flexural capacity becomes progressively more conservative as the concrete strength of the beam increases relative to that of the slab concrete and with increasing levels of tension reinforcement. This strength differential can lead to a situation where the strength limit state governs the design over the service limit state, which typically controls, and can affect load ratings.

The conservatism of the simplified approach can be attributed to neglecting the contribution of the beam top flange and its higher concrete strength to the resultant compression force. With increasing levels of beam reinforcement, the depth of the compression block increases to balance the tension force in the reinforcement, and the internal moment arm between the compression and tension resultant force decreases, leading to a lower estimate of flexural capacity than that calculated by the strain compatibility method. This reduced estimate of flexural capacity can, in turn, lead to sections that are predicted to be in the compression-controlled or transition region with flexural resistance factors ϕ less than 1.0. In these cases, strain compatibility analysis can provide a more accurate estimate of flexural resistance.

Simplified Approach

The simplified approach, based on a rectangular stress distribution and shown on the idealized beam section in **Fig. 1**, is well known in the engineering community. The basic premise is that the compression force in concrete can be obtained by assuming a rectangular stress distribution of magnitude $\alpha_1 f'_c$ acting over a portion ($a = \beta_1 c$) of the distance c between the extreme compression fiber and the neutral axis. The equations for estimating flexural capacity using this method are given in Article 5.6.2.2 of the AASHTO LRFD specifications with design compressive strengths for normalweight concrete up to 15.0 ksi and for lightweight concrete up to 10.0 ksi.

There are some important limitations to the simplified approach. In the AASHTO LRFD specifications, Commentary C5.6.2.2 and Article 5.6.3.2.6 specify that when the compression stress field includes elements of the cross section with different strengths of concrete, the lower-strength concrete is to be used in the equations of Articles 5.6.2 and

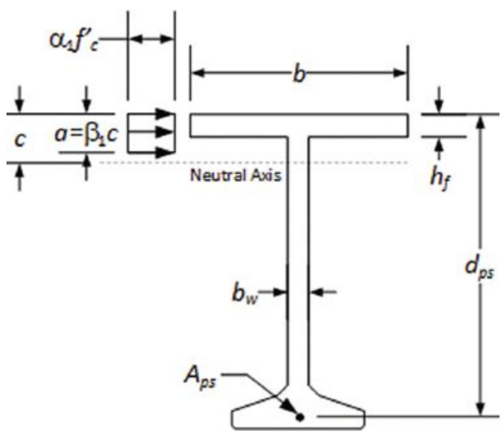

Figure 1. Idealized composite beam section for the AASHTO LRFD simplified, rectangular stress-block approach.
All Figures: Richard Brice.

5.6.3 for estimating flexural strength and other related parameters. A common misconception is that the lower-strength deck concrete can be transformed into an equivalent width of beam concrete using the modular ratio of the materials, as is done at the service limit state. This assumption is not valid at the strength limit state because strains are no longer in the linear-elastic range. Assumptions regarding the location and distribution of steel reinforcement are highlighted in Commentary C5.6.3.1.1. To use the equations, it must be reasonable to lump all the prestressing steel at a single location. However, it may not be reasonable to lump reinforcement spread vertically through the section, such as draped strands near end regions and post-tensioning tendons, into a single location. A similar assumption is made for nonprestressed tensile reinforcement. In sections where a significant number of prestressing elements are on the compression side of the neutral axis, C5.6.3.1.1 states that "it is more appropriate to use a method based on the conditions of equilibrium and strain compatibility as indicated in Article 5.6.2.1."

Strain Compatibility Approach

Article 5.6.3.2.5 of the AASHTO LRFD specifications recognizes the strain compatibility approach as an alternative to the simplified approach. Strain compatibility analysis involves subdividing the beam cross section into discrete parts or layers (as illustrated in Part 1 of this series), estimating the strain at the centroid of each layer based on the assumption of plane sections, evaluating the stress using stress-strain relationships, and integrating the stress field to determine the flexural strength of the section. This analysis is iterative until conditions of equilibrium are satisfied. The strain compatibility method can accurately analyze nonrectangular sections, concrete elements of varying strengths, and reinforcement at any position in the section.

Reinforcement strain limits can also be considered. The minimum properties of reinforcement are defined by their respective ASTM specifications. ASTM A416[2] defines the minimum properties for low-relaxation, seven-wire steel strand for prestressed concrete. The minimum elongation is specified as not less than 3.5% using a gauge length of not less than 24 in. Flexural capacity estimates should not rely on a strain greater than this value because the material is not required to provide a greater elongation before strand fracture occurs. In cases where concrete crushing at a strain of 0.003 is assumed and the equilibrium condition requires reinforcement strain greater than the minimum the material provides, the strain compatibility analysis should be repeated with strain in the reinforcement limited to 0.035. The resulting concrete compression strain will then be less than 0.003. This condition, which is easily accommodated with strain compatibility analysis, would negate a primary assumption of the simplified rectangular stress block approach.

The AASHTO LRFD specifications leave the details of the strain compatibility analysis, such as selection of appropriate stress-strain relationships for the concrete and reinforcement, to the engineering professional. Many stress-strain relationships for concrete have been developed, such as those by Hognestad[3] and Desayi and Kirshnan.[4] The bilinear stress-strain relationship used by El-Helou[5] and Tadros[6] is a simple and easy-to-use approximation. This bilinear model for concrete was used in the Part 1 example and is shown in **Fig. 2**. It is conservative, easily programmed into spreadsheet calculations, and amenable to hand calculations.

A bilinear elastic-plastic stress-strain relationship is often used for reinforcing bars and the well-known "power formula", presented in Part 1 of this article series, is typically used for prestressing strand.

Analysis Method Comparison

Consider a prestressed concrete beam with a composite cast-in-place deck slab with the dimensions, concrete strengths, and reinforcement presented in Part 1 of this series (**Fig. 3**). This section is analyzed with both the simplified and strain compatibility methods for increasing amounts of prestressing reinforcement, while keeping the center of gravity of the reinforcement constant for simplicity. The strain compatibility analysis uses the same bilinear model for the concrete stress-strain relationship (Fig. 2) and power formula for prestressing reinforcement from Part 1. **Figure 4** shows a comparison of the depth of the neutral axis and **Fig. 5** shows the comparison of factored flexural resistance for the two methods. **Figure 6** compares the stress in the prestressing reinforcement for each method.

The neutral axis depth is equal to the flange height when there is approximately 10 in.^{2} of prestressing reinforcement in the beam. Up to this amount of prestressing steel, only the slab is in compression and both methods result in essentially the same estimate of flexural capacity. Thereafter, the neutral axis depth, stress in the reinforcement, and flexural capacity diverge as the area of reinforcement increases.

At higher levels of reinforcement, the simplified method indicates that the section will enter the transition region between tension and compression-controlled behavior. At that point, the resistance factor for flexure becomes less than

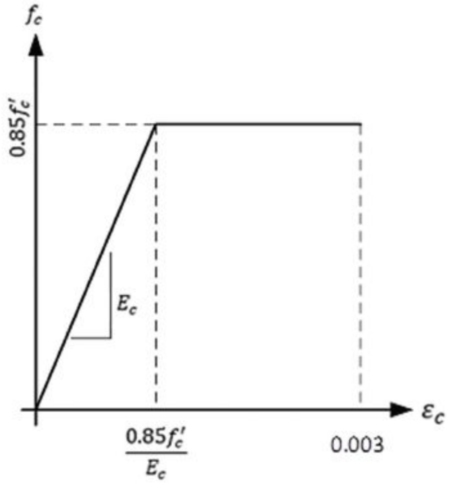

Figure 2. Bilinear stress-strain model for concrete.

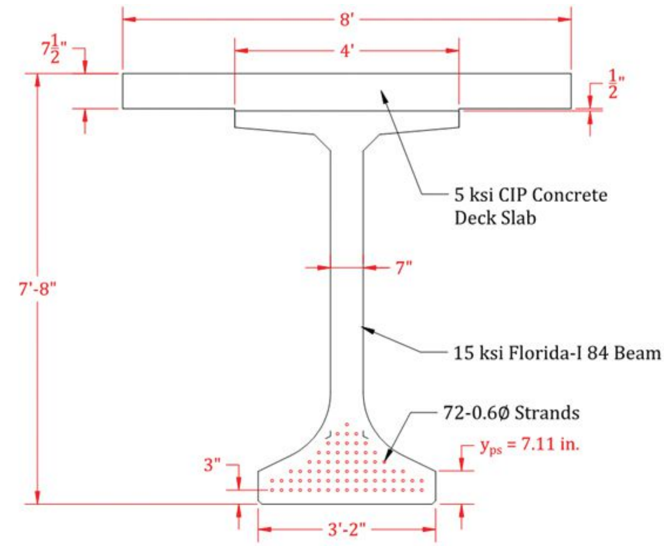

Figure 3. An 84-in.-deep Florida I-beam girder made composite with a 7.5-in.-thick, cast-in-place (CIP) structural deck slab used in the example analysis.

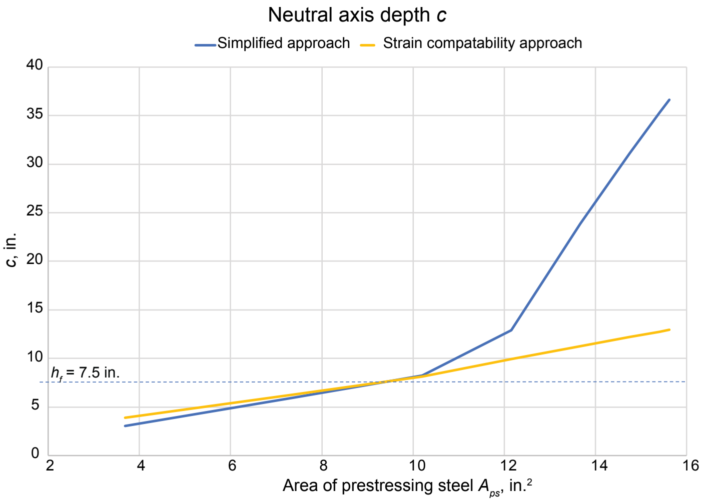

Figure 4. Depth of neutral axis comparison. Note: h_f = top flange thickness.

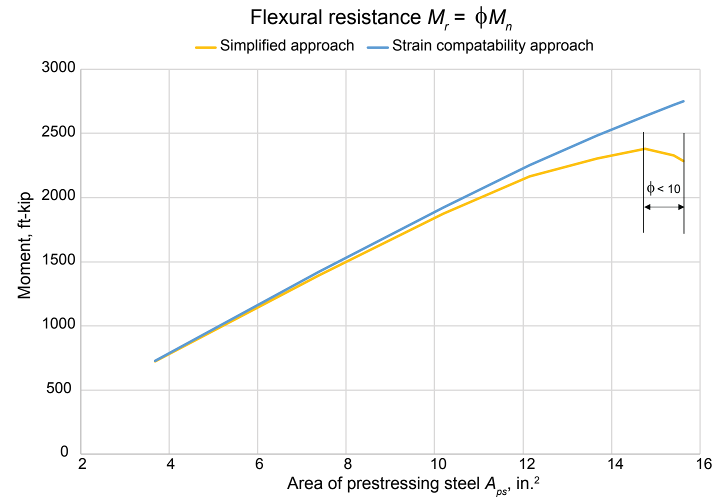

Figure 5. Comparison of flexural capacity.

1.0, increasing the conservatism of the estimated flexural capacity as compared with the strain compatibility analysis. The stress in the prestressing reinforcement decreases to less than the yield stress of 243 ksi for the simplified approach but remains significantly higher when the strain compatibility method is used.

The rectangular stress distribution–based simplified method has long been and remains an excellent method for estimating the flexural capacity of reinforced and prestressed concrete structural elements when the section analyzed conforms to the assumptions and limitations stated in the AASHTO LRFD specifications.

The simplified method becomes increasingly conservative because precast concrete has higher strength as compared with cast-in-place deck concrete, employs larger-diameter and higher-strength prestressing strands, and prestressed concrete beams with larger cross sections accommodate more strands, to the point where estimated flexural capacity at the strength limit state can control a design and limit load ratings.

The strain compatibility approach remedies this by providing more accurate estimates of flexural capacity that account for the size, shape, and properties of the prestressed concrete beam and the location and properties of the reinforcement.

References

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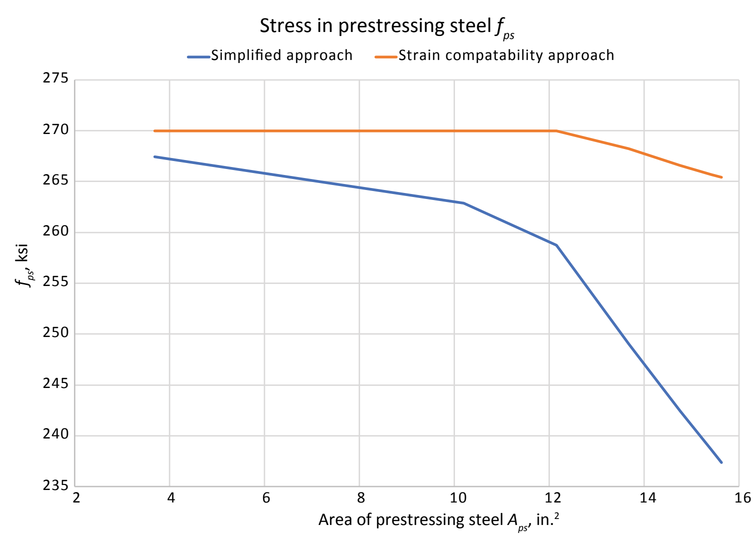

Figure 6. Comparison of stress in prestressing steel.