Application of the Wood-Armer Method for Slab Design

by Matthew C. Wagner, Colliers Engineering & Design

Concrete slabs are among some of the most common structural components in modern construction. In many cases, slab design can be approached with a simple one-way analysis, such as the strip method, where the slab is assumed to act as a beam element that is bending about its major axis oriented normal to the direction of the primary reinforcement. This approach has proven efficient and effective for most situations, where the slab geometry, boundary conditions, and loading are straightforward. However, in some scenarios—such as designs with curved alignments or skewed supports—this approach can yield unconservative results. Under such conditions, both direct bending moments and twisting moments should be considered in design to ensure adequate safety margins against overload and satisfactory in-service response.

This article reviews the mechanics of direct and twisting moments and provides an example application of the Wood-Armer method[1] in the design of the Bend Bridge, a curved, multispan, continuous, reinforced concrete slab bridge that provides pedestrian access from the Glass City Riverwalk to the Martin Luther King Bridge in Toledo, Ohio (**Fig. 1**). (For more information, see the Project article in the Summer 2025 issue of *ASPIRE*®.)

The Wood-Armer method, which was introduced by R. H. Wood in 1968, is one of the most popular rational design methods to incorporate the effect of twisting moments on the slab. Wood and G. S. T. Armer developed their approach from the normal moment yield criterion (also known as Johansen's yield criterion[2]), which aims to prevent yielding in the reinforcement in all directions. Their method combines direct bending and twisting actions into equivalent ultimate design moments that act normal to the primary reinforcement. The ultimate design moments for each reinforcement direction can be easily checked to ensure that they do not exceed the resisting moments.

Direct Bending Moments

Direct bending moments are derived from one-way slab action. They are the flexural forces that cause a slab to bend like a beam about a single axis.

- Direct bending moments M_{xx} and M_{yy} are flexure about the orthogonal slab *x*- and *y*-axes that result in out-of-plane displacement and curvature of the slab (**Fig. 2**).
- The out-of-plane displacement and curvature due to bending create linear strain profiles across the slab depth, generating compression on one face and tension on the opposite face. Reinforcement is required in regions of tensile stress.
- Direct or primary bending moments typically govern the slab's design for ultimate strength, crack control, and long-term deflection.

In conventional one-way slab designs, loads are distributed into strips and the resulting direct moments are checked against the resistance of the reinforcement. However, this approach ignores the twisting effects and resultant in-plane shear stresses that develop when a slab experiences two-way bending action.

Twisting Moments

A twisting moment M_{xy} arises when slab elements rotate about an in-plane axis due to shear forces acting on the slab surface. Description and derivation of the twisting moments can be found in any textbook on plate analysis.

- A twisting moment is effectively a torque within the plane of the slab.
- Unlike direct bending, twisting does not directly cause flexural cracking, but it induces in-plane shear stresses that reinforcement must balance.
- Twisting moments are particularly significant near corners, discontinuous edges, and skewed or curved support conditions, where high stress concentrations build concurrently in both primary axes.

Neglecting twisting moments can lead to inadequate reinforcement in high-stress

Figure 1. The Wood-Armer method was used for the design of the Bend Bridge, a curved, multispan, continuous, reinforced concrete slab bridge. Photo: Metroparks Toledo.

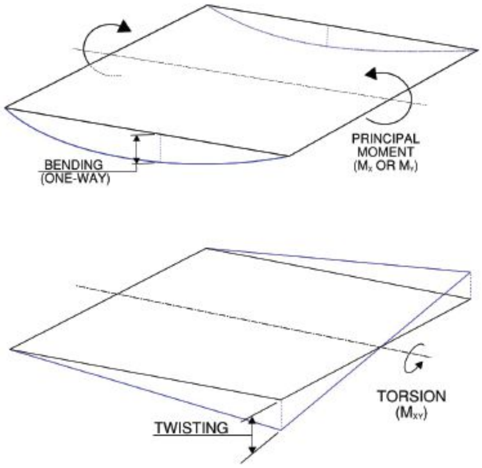


Figure 2. Basic bending and twisting moment diagrams. Figure: Colliers Engineering & Design.

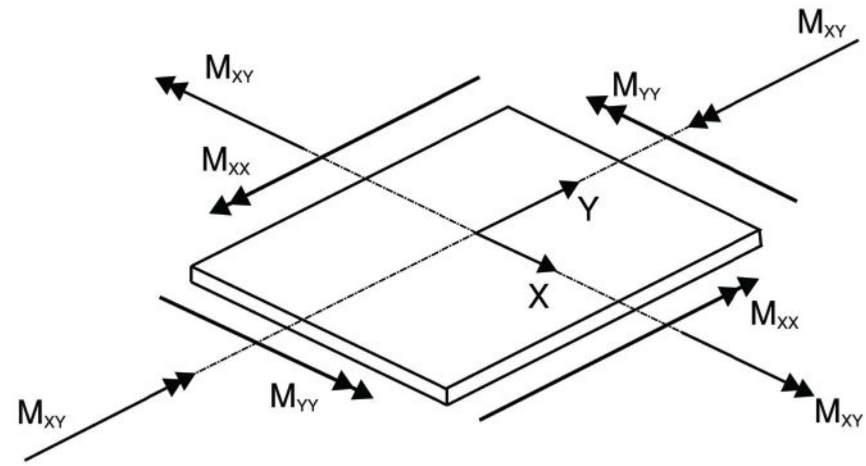


Figure 3. Notation and axis system used by R. H. Wood for plate direct bending and twisting moments. Figure: Colliers Engineering & Design.

regions, resulting in excessive cracking, serviceability concerns, and increased maintenance. Two-way slabs resist load through biaxial bending action (that is, loads are shared between orthogonal directions). The stress state results from a triad consisting of M_{xx}, M_{yy}, and M_{xy}.

- Biaxial action: Bending occurs about both the x and y axes, combined with twisting about the x-y plane.
- Load distribution: This interaction produces efficient structures but requires more sophisticated design than one-way strips.
- Reinforcement implications: Orthogonal reinforcement must simultaneously resist both direct and twisting contributions.

The challenge is to convert this triad into equivalent design moments for reinforcement, which is precisely the function of the Wood-Armer equations.

The Wood-Armer Method

Wood's paper "The Reinforcement of Slabs in Accordance with a Pre-determined Field of Moments" established a systematic approach for incorporating twisting moments into slab design. **Figure 3** shows the notation and axis system used by Wood for plate direct bending and twisting moments. The method, derived from Johansen's step yield-line criterion, has since been incorporated into many design guides, such as the American Concrete Institute's ACI 447R-18,[3] and finite element postprocessing tools. Some of the key principles of the method are as follows.

- Conversion of twisting moments: Twisting moments M_{xy} and direct bending moments are translated relative to the direction of the primary reinforcement for structural design.
- Design moments: For each slab face and primary reinforcement direction, the ultimate design moments are determined (**Fig. 4**).
- Efficient reinforcement layout: By resolving combined effects into orthogonal layers, the method avoids over- or underreinforcement for individual moment components.
- Applicability to skewed slabs: Extensions of the equations account for principal moment directions inclined relative to reinforcement axes, which facilitates application of the method to skewed bridge decks and irregular geometries.

Figure 5 shows example software output from the Bend Bridge design. For Wood-Armer moments, the reinforcement directions are defined relative to a reference axis:

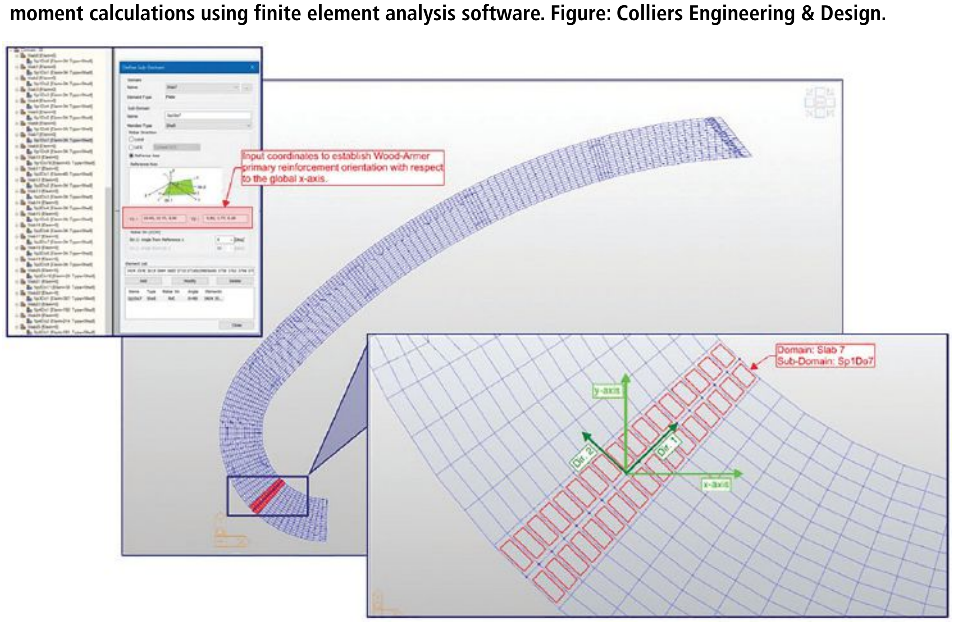
- W-A Moment, Top, Dir. 1 and W-A Moment, Bottom, Dir. 1 (defined as the slab top and bottom transverse reinforcement, respectively)
- W-A Moment, Top, Dir. 2 and W-A Moment, Bottom, Dir. 2 (defined as the slab top and bottom longitudinal reinforcement, respectively)

Integration with Finite Element Analysis

Most current finite element analysis (FEA) packages output M_{xx}, M_{yy}, and M_{xy} at nodes or elements. While these values represent the linear-elastic response of the plate or slab element, they cannot be used directly for reinforcement design unless the reinforcement is also aligned on the same coordinate system.

The Wood-Armer technique provides a rational method to link analysis and design, converting complex moment triads into ultimate design moments in the direction of the primary

Figure 4. Example of defining reinforcement directions relative to a reference axis for Wood-Armer moment calculations using finite element analysis software. Figure: Colliers Engineering & Design.



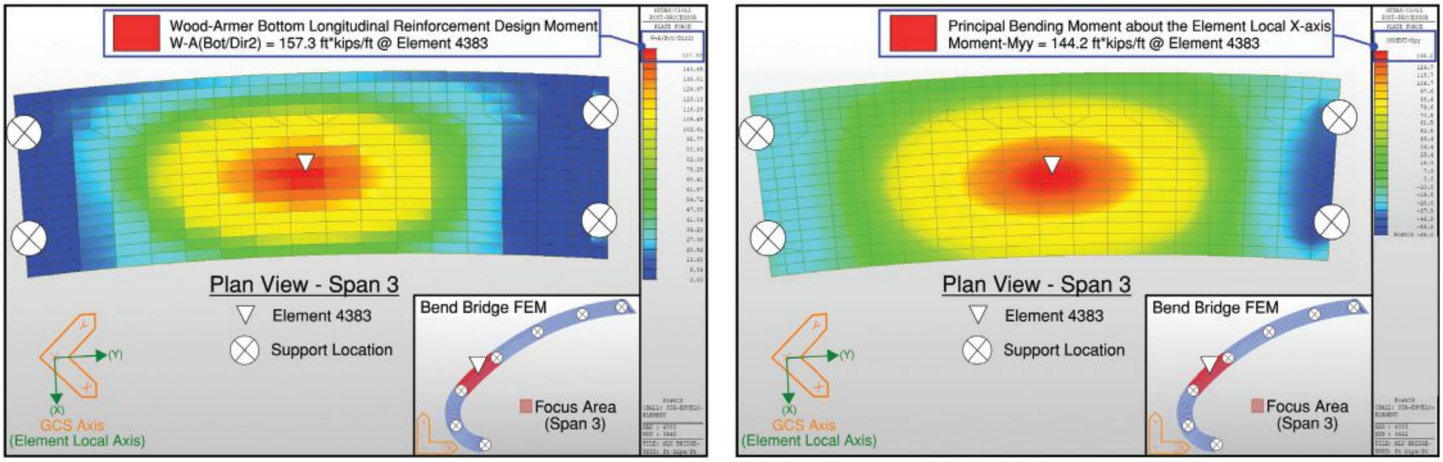


Figure 5. Comparison of Wood-Armer moments (left) and primary local bending moments (right) for the bottom longitudinal reinforcement design. If the primary local bending moment were used to design the bottom longitudinal reinforcement, the design would have been unconservative. Figure: Colliers Engineering & Design.

reinforcement and with respect to the location (top or bottom face) in the slab section. Without this step, there is potential for underreinforcement (if torsion is ignored) or overreinforcement (if moments are simply summed without redistribution).

Advantages of Wood-Armer Method with Finite Element Analysis

The use of the Wood-Armer method with FEA offers several benefits.

- Comprehensiveness: The method explicitly accounts for both direct bending and twisting moments that are neglected using strip-method design approaches.
- Design-oriented results: The output produces design moment contours aligned with the directions and locations of the primary reinforcement within the structural slab section.
- Efficiency: The method reduces unnecessary reinforcement by distributing twisting moments into equivalent bending moments.
- FEA design foundation: This technique forms the basis for slab reinforcement design modules in many current FEA software packages.

Limitations of Wood-Armer Method

Several caveats must be considered for this method:

- Manual complexity: Calculations are labor intensive if performed by hand. However, many software packages have integrated calculations that can be implemented directly.
- Assumptions: The method is based on linear elasticity and small deformations and does not capture nonlinear effects due to cracking.
- Structure type and material: The method is only applicable for reinforced concrete structural slabs.
- Potential for unconservative design: Research indicates that results may be unconservative in slabs with high reinforcement ratios (greater than 0.75%) under large torsion near restrained corners.
- Software output interpretation: Engineers must competently interpret contour plots and slab moments to avoid misapplication.

Final Thoughts

More than 50 years after its introduction, the Wood-Armer method remains relevant, particularly as analysis software advances. The base concepts are still relevant as a primary approach to break down complex analysis into an output format that engineers can understand and use for slab reinforcement design. When twisting moments are considered in addition to direct bending moments, the design and detailing of reinforcement, especially near corners and at discrete support points, are refined. The associated gains in serviceability and margins of safety demonstrate the power of using the Wood-Armer method for the design of structural slabs.

References

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The completed Bend Bridge in Toledo, Ohio, provides a pedestrian connection from the Glass City Riverwalk to the Martin Luther King Bridge in Toledo, Ohio. Figure: Kokosing Construction Co.